



# Evolving flexible beta basis function neural tree using extended genetic programming & Hybrid Artificial Bee Colony



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## ABSTRACT

In this paper, a new hybrid learning algorithm is introduced to evolve the flexible beta basis function neural tree (FBBFNT). The structure is developed using the Extended Genetic Programming (EGP) and the Beta parameters and connected weights are optimized by the Hybrid Artificial Bee Colony algorithm. This hybridization is essentially based on replacing the random Artificial Bee Colony (ABC) position with the guided Opposite-based Particle Swarm Optimization (OPSO) position. Such modification can minimize the delay which might be lead by the random position, in reaching the global solution. The performance of the proposed model is evaluated for benchmark problems drawn from time series prediction area and is compared with those of related methods.

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## 1. Introduction

The inefficiency of classical optimization algorithms to solve complex optimization problems such as highly non-linear problems encouraged researchers to employ natural or behavioral metaphors. This category of algorithms is called Evolutionary Computation (EC) [1]. Broadly speaking, evolutionary computing techniques include: evolution-inspired algorithms (such as: Genetic Algorithm [2], Differential Evolution [3]); swarm-inspired algorithms (such as: Ant Colony Optimization [4], Particle Swarm Optimization [5]); and other population-based algorithms such as Teaching–Learning-Based Optimization (TLBO) [6,7].

Recently Artificial Bee Colony (ABC) algorithm inspired of the bee behavior, was proposed by Karaboga in 2005 [8] for solving numerical optimization problems. Karaboga and Akay compared the ABC algorithm with GA, PSO, DE and Evolution Strategies (ES) optimization algorithms on large set of numerical test functions and results show that the performance of the ABC is competitive with that of other population-based algorithms [9]. The ABC algorithm is successfully used for many real-world problems. In fact, it

can efficiently be applied on training feed-forward neural networks [10,11] for pattern classification. Nandy et al. proposed, in Ref. [12], the ABC based back-propagation algorithm to optimize the back-propagation neural network training and hence the analysis of this method is performed over standard data sets, showing its efficiency in terms of convergence speed and rate. The ABC algorithm is also effectively used in energy field like the work presented by Rashidi et al. in Ref. [13] where the parametric study and optimization of regenerative Clausius and organic Rankine cycles with two feed-water heaters were achieved basing on artificial neural network and ABC algorithm. This ABC algorithm was applied to medical pattern classification and clustering problems [14,15] and to solve TSP problems [16].

Although ABC algorithm is an efficient algorithm for many optimization problems, there still have several weaknesses, such as it easily gets stuck in the local optima for some complex multimodal problem and its random candidate food position may lead to delay in reaching the global solution [9]. These issues encourage some researchers to hybridize it with other algorithms such as the hybridization with DE algorithm proposed by Kong et al. [17]; and the hybridization with PSO algorithm introduced by Shi et al. [18] for global numerical optimization. In the present work, a hybridization between ABC algorithm and Opposite-based PSO [19,57,58] algorithm, called ABC-OPSOP, is proposed to optimize the Beta Basis Function Neural Network (BBFNN) parameters.

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In fact, a BBF neural network’s performance depends mainly on two issues which are the design of the network structure and the architecture parameter’s adjustment automatically. In order to learn connected weights and Beta parameters which include the center, width and form parameters of BBFNs, many methods are applied, i.e., back-propagation algorithm [20], genetic algorithm [21], DE algorithm [22], PSO algorithm [23] and so on. In addition, many important attempts have been developed to optimize both structure and parameters of the BBFNN such as Hierarchical Genetic Algorithm (HGA) [24] and Hierarchical Multi-dimensional Differential Evolution (HMDDE) [25].

Although conventional representation of BBFNN has a number of advantages such as better approximation capabilities and simple network topologies, however adapting the matrix-representation suffers from slow premature convergence characteristics and makes the BBFNN’s structure difficult to regulate. These reasons motivate us to apply the tree-based encoding method which was introduced by Chen et al. [26–28], to represent a BBFNN. The new model is called flexible beta basis function neural tree (FBBFNT) [29,42,53,54].

In this paper, the FBBFNT model is applied to benchmark problems drawn from time series prediction area. Based on the predefined Beta operator sets, a flexible beta basis function neural tree model can be created and evolved. The hierarchical structure is evolved using the Extended Genetic Programming (EGP). The fine tuning of the Beta parameters (centre, spread and the form parameters) and weights encoded in the structure is accomplished using an Artificial Bee Colony algorithm modified by Opposite-based PSO positions (ABC-OPSOP). Experimental results and comparisons demonstrate the effectiveness and efficiency of the proposed ABC-OPSOP algorithm.

The remainder of this paper is organized as follows: Section 2 describes the basic flexible beta basis function neural tree. A hybrid learning algorithm for evolving the Beta function neural tree model is the subject of Section 3. The set of some simulation results are provided in Section 4. Finally, some concluding remarks are presented in Section 5.

### 2. Flexible beta basis function neural tree representation

In 1997, Alimi [30] proposed the use of Beta basis function as transfer function for training Beta Basis Function Neural Network. The Beta basis function has many advantages such as its large flexibility, its universal approximation characteristics [31] and its ability to generate rich shapes (asymmetry, linearity, etc.) [32].

In this work, a tree-based encoding method is adopted to represent the Beta basis function neural network instead of the matrix-based encoding, as it is more flexible and gives a more adjustable and modifiable architecture. In fact, matrix representation has limited flexibility in expressing topologies of the network structure with variable layers. Thereafter, evolutionary computation operators need to be applied carefully to preserve the topological constraints of networks. This representation is called flexible beta basis function neural tree (FBBFNT).

The FBBFNT is formed of a node set  $NS$  representing the union of function node set  $F$  and terminal node set  $T$ :

$$NS = F \cup T = \{\beta_2, \beta_3, \dots, \beta_N, /N\} \cup \{x_1, \dots, x_M\} \quad (1)$$

where

- $\beta_n$  ( $n=2, 3, \dots, N$ ) denote non-terminal nodes and represent flexible Beta basis neurons with  $n$  inputs and  $N$  is the maximum degree of the tree.
- $/N$  is the root node and represent a linear transfer function.
- $x_1, x_2, \dots, x_M$  are terminal nodes and define the input vector values.

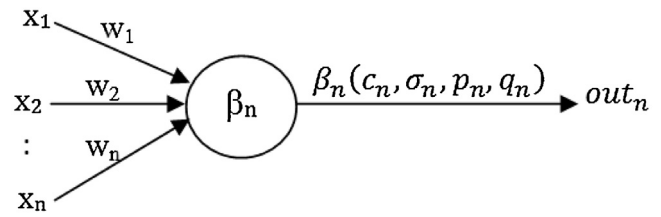


Fig. 1. A flexible neuron Beta operator.

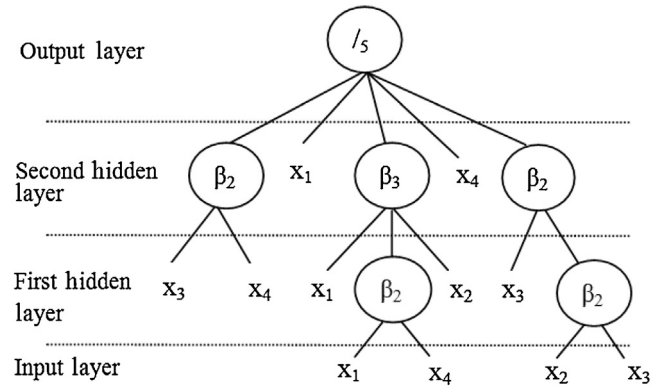


Fig. 2. A typical representation of FBBFNT: function node set  $F = \{\beta_2, \beta_3, \beta_4, \beta_5, /5\}$ , and terminal node set  $T = \{x_1, x_2, x_3, x_4\}$ .

The output of a non-terminal node is calculated as a flexible neuron model (see Fig. 1).

If a function node, i.e.,  $\beta_n$  is selected,  $n$  real values are randomly created to represent the connected weight between the selected node and its offspring. In addition, seen that the flexible activation function used for the hidden layer nodes is the Beta function, four adjustable parameters (the center  $c_n$ , width  $\sigma_n$  and the form parameters  $p_n, q_n$ ) are randomly generated as flexible Beta operator parameters.

For each non-terminal node, its total excitation is calculated by:

$$y_n = \sum_{j=1}^n w_j \times x_j \quad (2)$$

where  $x_j$  ( $j=1, \dots, n$ ) are the inputs of the selected node and  $w_j$  ( $j=1, \dots, n$ ) are the connected weights.

The output of node  $\beta_n$  is then calculated by:

$$out_n = \beta_n(y_n; c_n, \sigma_n, p_n, q_n) = \begin{cases} \left[ 1 + \frac{(p_n + q_n)(y_n - c_n)}{\sigma_n p_n} \right]^{p_n} \left[ 1 + \frac{(p_n + q_n)(c_n - y_n)}{\sigma_n q_n} \right]^{q_n} \\ \text{if } y_n \in ]c_n - \frac{\sigma_n p_n}{p_n + q_n}, c_n + \frac{\sigma_n q_n}{p_n + q_n}[ \\ \text{0 else} \end{cases} \quad (3)$$

The output layer yields a vector by linear combination of the node outputs of the last hidden layer to produce the final output.

A typical flexible beta basis function neural tree model is shown as Fig. 2. The overall output of flexible beta basis function neural tree can be computed recursively by depth-first method from left to right.

### 3. The hybrid FBBFNT evolving algorithm

The optimization of FBBFNT includes the tree-structure and parameter optimization. In this study, finding an optimal or a near optimal Beta basis function neural tree structure is achieved by using Extended Genetic Programming algorithm (EGP) and the

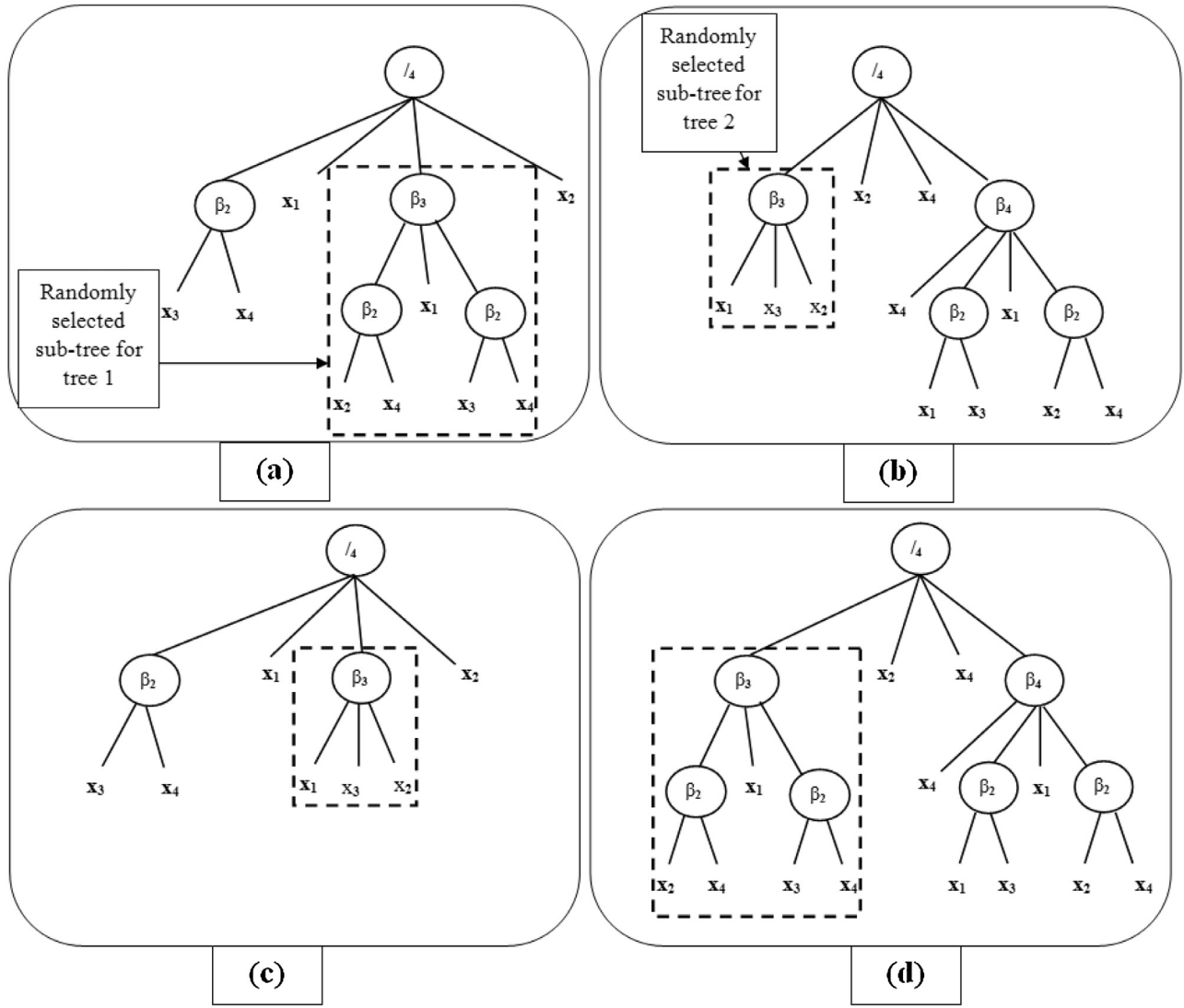


Fig. 3. Examples of the EGP crossover operator: (a) selected tree 1. (b) Selected tree 2. (c) Tree 1 after crossover operator with tree 2. (d) Tree 2 after crossover operator with tree 2.

parameters implanted in a FBBFNT are optimized by Artificial Bee Colony algorithm modified by Opposite-based PSO positions (ABC-OPSOP).

### 3.1. Structure optimization

The first step of the structure optimization is to create an initial population of flexible Bata basis function trees with random structures; i.e., uniformly distributed random number of layers in  $[3, NL\_Max]$  and uniformly distributed random number of nodes for each layer in  $[2, NN\_Max]$ , where  $NL\_Max$  (the maximum layer number) and  $NN\_Max$  (the maximum node number) are chosen depending on the studied problem. The node parameters (Beta parameters and connected weights) of each tree are also randomly generated in its search spaces. Indeed, the search space of each node parameter is its interval limits:  $[Min_c, Max_c]$  for  $c$ ,  $[Min_\sigma, Max_\sigma]$  for  $\sigma$ ,  $[Min_p, Max_p]$  for  $p$ ,  $[Min_q, Max_q]$  for  $q$  and  $[0, 1]$  for  $w$ . Each individual is then evaluated according to the structure fitness function.

In the second time, a number of flexible neural tree variation operators, which are an extension of standard Genetic Programming (GP).

#### 3.1.1. Structure fitness function

According to Zhang and Mühlenbein [35] and many other researches [24,25], the fitness function adopted to find an optimal or a near-optimal Artificial Neural Network structure, depends mainly on both the ANN performance on the training data (such as the error and the training time or the number of functions evaluation) and the ANN complexity measurement (such as the number of nodes and the number of layers). The structure objective function to minimize which we used in our study is composed so of two measurements  $f_1$  and  $f_2$ . The function  $f_1$  measures the Root Mean Squared Error (RMSE) between the target and output of the proposed model. The function  $f_2$  measures the complexity of the FBBFNT model.

$$Fit_{stru}(i) = \alpha f_1(i) + \delta f_2(i) \tag{4}$$

where  $\alpha, \delta$  are user specified fitness coefficients that allow a trade-off between the objectives,  $\alpha, \delta \in [0, 1]$ .

The function  $f_1(i)$  denoted the RMSE value of  $i$ -th individual, is as follows.

$$f_1(i) = \sqrt{\frac{1}{P} \sum_{j=1}^P (y_t^j - y_{out}^j)^2} \tag{5}$$

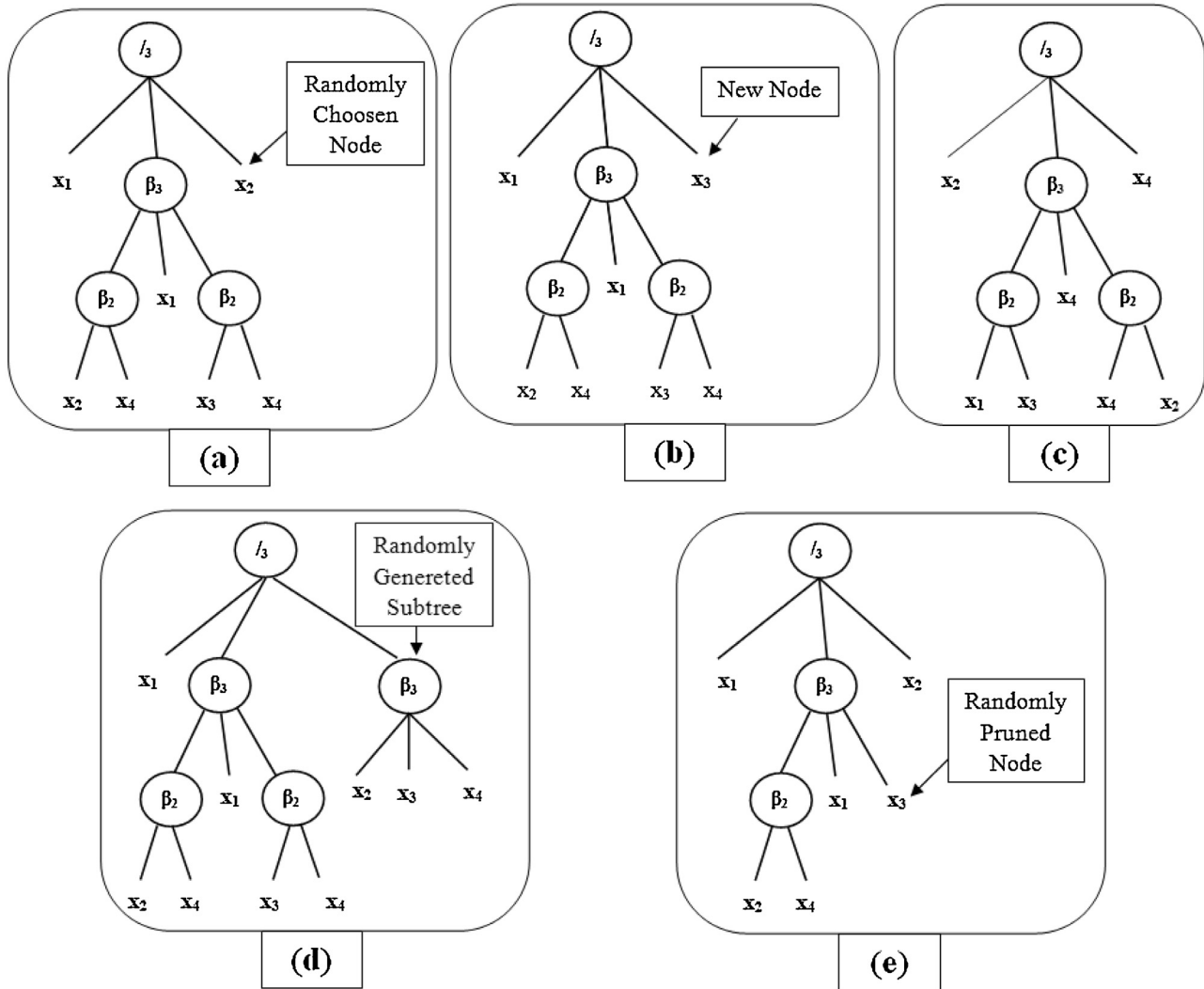


Fig. 4. Examples of the four EGP mutation operators: (a) original tree. (b) Changing one leaf node. (c) Changing all the leaf nodes. (d) Growing. (e) Pruning.

where  $P$  is the total number of samples,  $y_t^j$  and  $y_{out}^j$  are the desired output and the FBBFNT model output of  $j$ -th sample.

After a series of tuning experiments of the expression of  $f_2(i)$  based on the number of structure nodes, the used expression is as follows.

$$f_2(i) = \frac{Size_i - SizeMin + 1}{SizeMax - Size_i + 1} \quad (6)$$

where  $Size_i$  is the node number of the  $i$ -th individual.  $SizeMin$  is equal to 5 and  $SizeMax$  can be calculated as following:

$$SizeMax = \frac{NNMax^{NNMax} - 1}{NNMax - 1} \quad (7)$$

### 3.1.2. Extended Genetic Programming: EGP

The Extended Genetic Programming which is an extended version of the standard genetic programming is formed by three mainly operators.

**3.1.2.1. Selection operator.** Its purpose mainly is to select two parents from the population in order to procreate a new child by crossover or mutation operator. In this study firstly a truncation selection is used by ranking all individuals according to their fitness. After that, a threshold  $T$  ( $T \in [0, 1]$ ) is applied such that the  $(1-T)\%$  best individuals are selected to survive to the next genera-

tion and the remaining individuals are removed and replaced with new ones. Secondly, the binary tournament selection is also used to select individuals for the crossover or mutation operator. For each individual, two opponents are randomly chosen from all the parents and offspring. For each comparison, if the fitness of individual exceeds that of the opponent, it receives a selection.

**3.1.2.2. Crossover operator.** Its principle is to take randomly selected sub-trees in the individuals and to select randomly one non-leaf node in the hidden layer for each chromosome, and then swapping the selected sub-trees. An example of crossover operator is shown in Fig. 3.

**3.1.2.3. Mutation operators.** Four different mutation operators were used to generate offspring from the parents:

1. *Changing one leaf node:* select one leaf node randomly in the FBBFNT and replace it with another leaf node (Fig. 4(b));
2. *Changing all the leaf nodes:* select each leaf nodes in the FBBFNT and replace it with another leaf node (Fig. 4(c));
3. *Growing:* select a random leaf node in hidden layer of the FBBFNT and replace it with a randomly generated sub-tree (Fig. 4(d));
4. *Pruning:* randomly select a Beta operator node in the FBBFNT and replace it with a random leaf node (Fig. 4(e)).

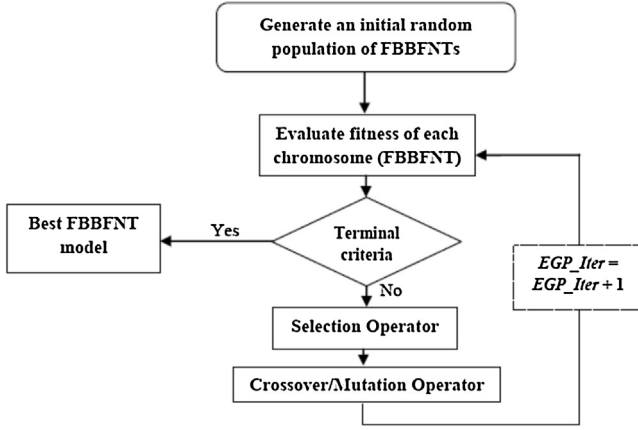


Fig. 5. Flowchart of the Extended Genetic Programming algorithm.

The EGP mutation operators were applied to each parent according to the method of Chellapilla [34] as following:

- Define a number  $M$  which represents a sample from a Poisson random variable.
- Select randomly  $M$  mutation operators from above four mutation operator set.
- Apply these  $M$  mutation operators in sequence one after the other to the parent to create the offspring.

The new generation which modify some individuals of the population, is evaluated using the structure fitness function. If this generation is better than the previous one, the population is updated. This process can be repeated until the terminal criteria are reached; i.e., a better structure is found or a maximum number of EGP iterations is attained. A schematic overview of EGP is given in Fig. 5.

### 3.2. Parameter optimization

After obtaining the optimal or near-optimal structure of the FBBFNT model using the Extended Genetic Programming (EGP), the hybrid system move to improve the performance of the parameters (weights and Beta parameters) encoded in this best found structure.

To find optimal or near-optimal FBBFNT parameters, the first step is to decide the corresponding representation. In our case, we used the real-number representation for mapping the FBBFNT parameters into a matrix-representation. Indeed, the real-number representation is preferred compared to the binary-string representation especially when the number of connection weights is very large and the desired error is very small.

The second step is the choice of the learning method. In this context several researches have been achieved and most of all proved that Evolutionary Computation (EC) gives better results compared to classical methods. For these reasons, we adopted, as learning algorithms, existing or hybrid evolutionary computation algorithms. The parameter fitness function used to evaluate individuals will be presented in the next section.

#### 3.2.1. Parameter fitness function

To find an optimal FBBFNT, the Root Mean Squared Error (RMSE) is employed as a fitness function:

$$\text{Fit}_{\text{par}}(i) = \sqrt{\frac{1}{P} \sum_{j=1}^P (y_t^j - y_{\text{out}}^j)^2} \quad (8)$$

where  $P$  is the total number of samples,  $y_t^j$  and  $y_{\text{out}}^j$  are the desired output and the FBBFNT model output of  $j$ th sample.  $\text{Fit}_{\text{par}}(i)$  denotes the fitness value of  $i$ th individual.

#### 3.2.2. Artificial Bee Colony algorithm

Artificial Bee Colony algorithm (ABC) is a swarm intelligent optimization algorithm based on the metaphor foraging behavior of honey bee swarm, proposed by Karaboga in 2005 [8]. This model that leads to the emergence of collective intelligence of honey-bee swarms consists of three essential bee kinds: employed bees, onlooker bees and scout bees. Employed bees are responsible for exploiting a specific food source and sharing the information of the nectar amount with onlooker bees waiting in the hive.

Onlooker bees watch the dances of employed bees and choose food sources depending on the quantity information. Scout bees search the space randomly to find a new nectar source. The number of employed bees is equal to the number of nectar sources. The position of a nectar source represents a possible solution to the optimization problem, and the nectar amount of a food source corresponds to the fitness of the associated solution.

Denote the food sources number as  $NP$ . In our case, the position of the  $i$ th food source as a chromosome  $x_i$  ( $i = 1, \dots, NP$ ) which is  $NP_{\text{Param}} \times NN$  matrix; with  $NP_{\text{Param}}$  is the number of parameters (Beta parameters and connected weights) and  $NN$  is the number of FBBFNT nodes. A candidate food position from the old one in memory can be generated as:

$$v_i = x_i + \varphi_1 (x_i - x_k) \quad (9)$$

where  $k \in \{1, \dots, NP\}$  with  $k \neq i$  and  $\varphi_1$  are randomly distributed number in  $[0,1]$ . Then,  $v_i$  is compared with  $x_i$ , and the better one should be remained.

An onlooker bee chooses a food source depending on the probability value associated with that food source, which is given by:

$$p_i = \frac{\text{Fit}_{\text{par}}(i)}{\sum_{j=1}^{NP} \text{Fit}_{\text{par}}(j)} \quad (10)$$

where  $\text{Fit}_{\text{par}}(i)$  is the fitness value of the  $i$ th food source.

If the abandoned food source is  $x_i$ , the scout bee discovers a new food source according to:

$$x_i = a_j + \text{rand}_i (b_j - a_j) \quad (11)$$

where  $[a_j, b_j]$  is the search space of each parameter,  $j \in \{1, \dots, NP_{\text{Param}}\}$ .

#### 3.2.3. Opposite-based Particle Swarm Optimization (OPSO)

PSO was proposed by Kennedy and Eberhart [36] and is inspired by the swarming behavior of animals. The initial population of particles is randomly generated. Each particle has a position vector denoted by  $x_i$ . A swarm of particles 'flies' through the search space; with the velocity vector  $v_i$  of each particle. At each time step, a fitness function is calculated by using  $x_i$ .

Each particle records its best position corresponding to the best fitness, which has done so far, in a vector  $p_i$ . Moreover, the best position among all the particles obtained in a certain neighborhood of a particle is recorded in a vector  $p_g$ .

The use of heuristic operators or the update of the position in the PSO algorithm can mislead the finding of best particle by heading it towards a bad solution. Consequently, the convergence to the desired value becomes very expensive. To avoid these drawbacks, research dichotomy is adapted to improve the generalization performance and accelerate the convergence rate. Thus the reduction of the convergence time of the beta neural system is done by dividing the search space in two subspaces and a concept of the opposite number can be used to look for the guess solution between the two search subspaces. This concept can be integrated in the basic PSO

algorithm and form a new algorithm called Opposite-based Particle Swarm Optimization (OPSO) [19,57,58].

The OPSO steps are described as follows:

**Step 0 (Initialization):** At iteration  $t=0$ , the initial positions  $x_i(0)$  ( $i=1, \dots, NP$ ) which are  $NParam \times NN$  arrays, are generated uniformly distributed randomly as Eq. (11);

Generate the opposite population as follows:

$$\bar{x}_i(0) = \begin{cases} \alpha_i(x_i(0) + \frac{a_j + b_j}{2}), & \text{if } x_i(0) < \frac{a_j + b_j}{2} \\ \alpha_i(x_i(0) - \frac{a_j + b_j}{2}), & \text{if } x_i(0) > \frac{a_j + b_j}{2} \end{cases}, \alpha_i \in ]0, 1[ \quad (12)$$

The initial velocities,  $v_i(0)$ ,  $i=1, \dots, NP$ , of all particles are randomly generated.

**Step 1 (Particle evaluation):** Evaluate the performance of each particle in the population using a fitness function (Section 3.2.1).

**Step 2 (Velocity update):** At iteration  $t$ , the velocity  $v_i$  of each particle  $i$  is updated using  $p_i(t)$  and  $p_g(t)$ .

Here, the mutation operator is adopted according to:

$$v_i(t+1) = \Psi(t)v_i(t) + c_1\varphi_1(p_i(t) - x_i(t)) + c_2\varphi_2(p_g(t) - x_i(t)) \quad (13)$$

where  $c_1, c_2$  (acceleration) and  $\Psi$  (inertia) are positive constant and  $\varphi_1$  and  $\varphi_2$  are randomly distributed number in  $[0,1]$ . The velocity  $v_i$  is limited in  $[-v_{max}, +v_{max}]$ .

**Step 3 (Position update):** Depending on their velocities, each particle changes its position and its opposite- position according to:

$$x_i(t+1) = x_i(t) + (1 - \Psi(t))v_i(t+1) \quad (14)$$

$$\bar{x}_i(t+1) = \bar{x}_i(t) + (1 - \Psi(t))v_i(t+1) \quad (15)$$

**Step 4 ( $p_i$  and  $p_g$  update):** After traveling the whole population and changing the individual positions, the values of  $p_i(t)$  and  $p_g(t)$  obtained so far are updated.

### 3.2.4. Artificial Bee Colony algorithm modified by Opposite-based Particle Swarm Optimization positions (ABC-OPSOP)

In ABC, the best solution found so far is not always held in the population unlike PSO and OPSO. Since, it might be replaced, at some limit, with a randomly produced solution by a scout. Therefore, it might not contribute to the creation of trial solutions. Moreover, ABC algorithm suffers from slower convergence speed for some unimodal problems and easily gets trapped in the local optima for some complex multimodal problems.

For these reasons, we proposed to hybridize the classical ABC with the Opposite-based Particle Swarm Optimization (OPSO). This hybridization is essentially based on replacing the solution search equation of ABC algorithm (Eq. (9)) by the position equations of the OPSO algorithm (Eqs. (14) and (15)). The solution procedure of ABC-OPSOP is as follows:

### ABC-OPSOP algorithm

1: Initialize the food sources and evaluate the population and the opposite population (step 0 and step 1 of OPSO algorithm),  $trail_i = 0$ , ( $i = 1, \dots, NP$ ),  $cycles=1$ ,  $NFEs$ = number of Function Evaluations of EGP.

2: Repeat

3: Produce the solution  $x_i$  according to (14) and the opposite solution  $\bar{x}_i$  according to (15) for employed bees and evaluate them.

4:  $NFEs = NFEs + 2$

5: Select the better solution between these two solutions using the fitness value.

6: If solution does not improve  $trail_i = trail_i + 1$ , otherwise  $trail_i = 0$ .

7: Calculate the probability according to (10) and apply roulette wheel selection scheme to choose a food source for onlooker bees.

8: Produce the solution  $x_i$  according to (14) and the opposite solution  $\bar{x}_i$  according to (15) for onlooker bees and evaluate them.

9:  $NFEs = NFEs + 2$

10: Select the better solution between these two solutions using the fitness value.

11: If solution does not improve  $trail_i = trail_i + 1$ , otherwise  $trail_i = 0$ .

12: Update  $p_i(cycles)$  and  $p_g(cycles)$  values.

13: If  $(max(trail_i) > limit)$

replace this food source with a new randomly produced food source by (11) and evaluate it  $NFEs = NFEs + 1$

14: Memorize the best solution achieved so far.

15:  $cycles=cycles+1$ .

16: Until (Terminal criteria satisfied)

### 3.3. Learning algorithm for FBBFNT model

To find an optimal or near-optimal FBBFNT model, structure and parameters optimization are used alternately. Combining the EGP and ABC-OPSOP algorithms, a hybrid algorithm for evolving FBBFNT model is described as follows and is depicted in Fig. 6. The FBBFNT model is also freely available as Matlab code [62].

(a) Create randomly an initial population (FBBFNT trees and its corresponding parameters);

$G=0$ , where  $G$  is the generation number of the learning algorithm;

$global\ iterations = 0$ ;

$NFEs = 0$ ;

(b) Structure optimization is accomplished by the Extended Genetic Programming (EGP) as described in Section 3.1;

(c) If the terminal criteria are reached, then go to step (d),

$global\ iterations = global\ iterations + EGP\ iterations$ ;

$NFEs = NFEs + number\ of\ Function\ Evaluations\ of\ EGP$ ;

otherwise go to step (b);

(d) Parameter optimization is achieved by the ABC-OPSOP algorithm. The architecture of FBBFNT model is fixed, and it is the best tree found by the structure search. The parameters (weights and flexible Beta function parameters) encoded in the best tree formulate a food source;

(e) If the terminal criteria is satisfied, or no better parameter vector is found for a fixed time then go to step (f),

$global\ iterations = global\ iterations + cycles$ ;

$NFEs = NFEs + number\ of\ Function\ Evaluations\ of\ ABC-OPSOP$ ;

otherwise go to step (d);

(f) If satisfactory solution is found or a maximum global iteration number is reached, then the algorithm is stopped; otherwise let  $(G = G + 1)$  and go to step (b);

## 4. Experimental results

Nonlinear problems are of interest to scientists and engineers because most physical systems are inherently nonlinear in nature.

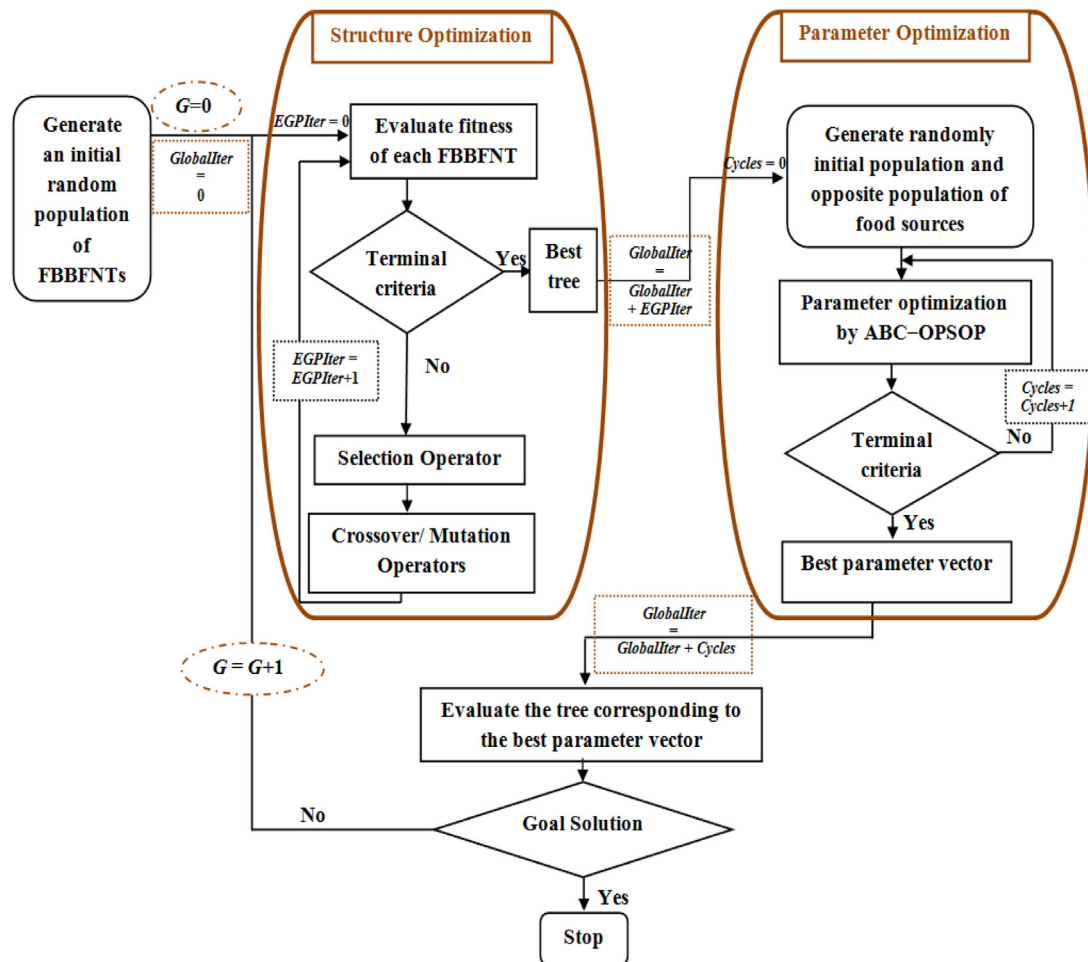


Fig. 6. The Hybrid Learning Algorithm for FBBFNT model.

In addition, nonlinear equations are difficult to solve and give rise to important subsystems such as chaotic, identification and control systems.

Chaotic systems which have held researchers' interest in the past decades, is one of the most used nonlinear systems for the artificial neural network test. As well-known benchmarks of chaotic systems, we can note the time-series problems. These problems are based on the following principle: knowing the solution at recent past values of the observed series, we can predict the solution at a point in the future. According to the literature, the widely used examples are the Mackey-Glass, Jenkins-Box, sunspot number, and Lorenz chaotic time series.

Another class of nonlinear systems that is broadly used in the literature consisted of the nonlinear plant identification problems. In fact, the identification system is model of a system which can be represented by a mathematical model based in the physical laws that govern the problem or by using the experimental data measured in the own system [59].

In the identification context, the methods do not need previous knowledge of the system, they are so known as black-box identification process [60].

Therefore, we apply in this section, the proposed FBBFNT model to predict and identify these nonlinear systems.

After a series of tuning experiments of the system settings, the chosen setting values for the structure optimization algorithm and the parameter optimization algorithm are as listed in Table 1. These FBBFNT settings are the same for all problems. For all examples the illustrated results are obtained by averaging the results in 30

Table 1  
FBBFNT parameters.

|                 | Parameter                 | Initial value |
|-----------------|---------------------------|---------------|
| EGP             | Population size           | 50            |
|                 | Crossover probability     | 0.3           |
|                 | Mutation probability      | 0.6           |
|                 | Generation gap            | 0.9           |
|                 | Maximum generation number | 4000          |
| PSO             | $c_1$                     | 0.2           |
|                 | $c_2$                     | 0.2           |
| OPSO            | $c_1$                     | 0.8           |
|                 | $c_2$                     | 0.8           |
| ABC             | ABC.LimitTrial            | 50            |
| ABC-OPSOP       | Population size           | 50            |
|                 | Maximum iteration number  | 4000          |
|                 | $c_1$                     | 0.8           |
|                 | $c_2$                     | 0.8           |
|                 | ABC.LimitTrial            | 50            |
| Common settings | Population size           | 50            |
|                 | Maximum iteration number  | 4000          |

runs. To measure the time complexity of the hybrid algorithm, we adopted the number of Function Evaluations: NFEs.

We used Matlab environment using a PC with Intel(R) Core(TM) i3-2370 M CPU 2.4 GHz and 4GB RAM.

Several tests will be used to compare the effectiveness of all competitor algorithms with respect to three measures of performance which are:

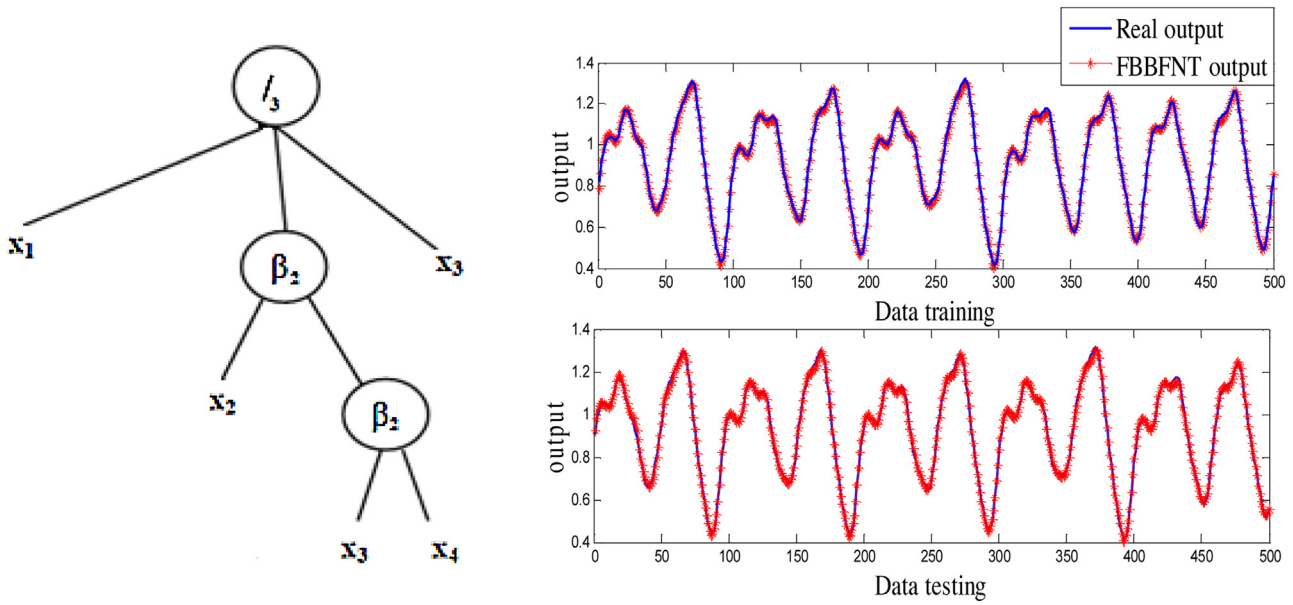


Fig. 7. The evolved FBBFNT (left), and the actual time series data and the output of the FBBFNT model for training and test samples (right) to forecast Mackey–Glass data.

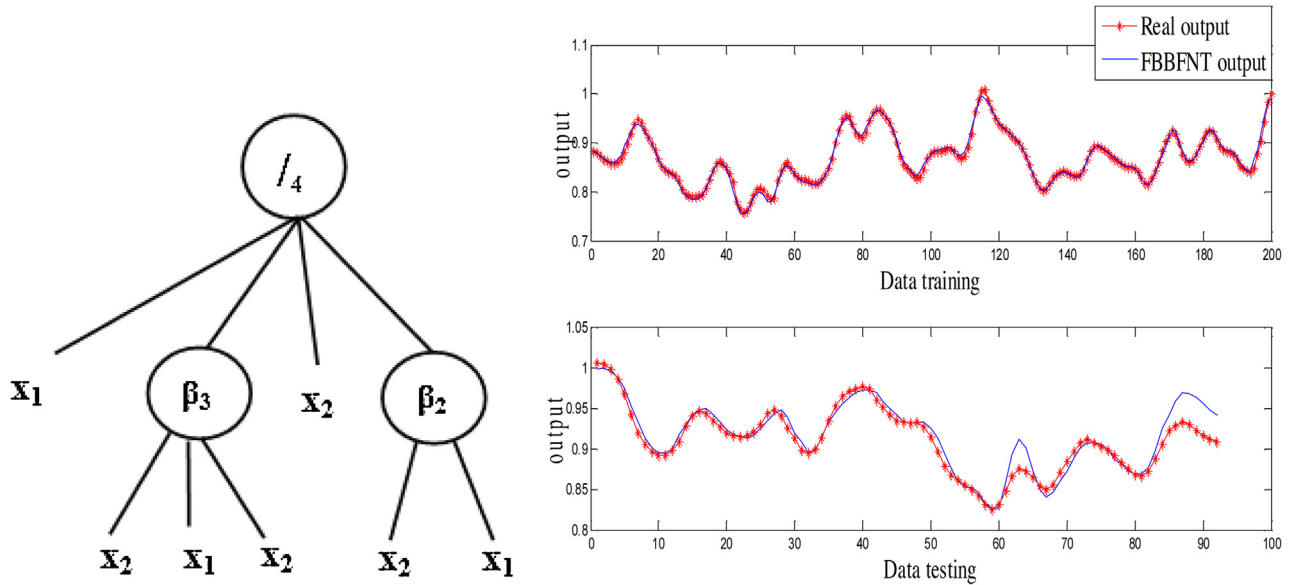


Fig. 8. The evolved FBBFNT (left), and the actual time series data and the output of the FBBFNT model for training and test samples (right) to predict the Jenkins–Box time-series ( $y(t - 1), u(t - 4)$ ).

- Solution quality or accuracy: by calculating the RMSE values,
- Convergence speed: through the Global Iteration Number (GIN),
- Complexity: FBBFNT complexity (number of function hidden nodes) and hybrid algorithm complexity (number of Function Evaluations: NFEs).

In addition, we will consider other performance characteristics of the model, namely, the memory footprint of the source code and the input data in *kilo-bytes* as well as the execution time in *seconds*.

The developed software was also tested on other datasets and applications but due to space limitations, we are not able to include all the results in this manuscript.

#### 4.1. Example 1: Mackey–Glass time series prediction

A time-series prediction problem can be constructed based on the Mackey–Glass [37] differential equation:

$$\frac{d(x(t))}{dt} = \frac{ax(t - \tau)}{1 + x^c(t - \tau)} - bx(t) \tag{16}$$

The setting of the experiment varies from one work to another. In this work, the same parameters of Refs. [18,19], and [22] namely  $a = 0.2, b = 0.1, c = 10$  and  $\tau \geq 17$ , were adopted, since the results from these works will be used for comparison. As in the studies mentioned above, the task of the neural network is to predict the value of the time series at point  $x(t + 6)$ , with using the inputs variables  $x(t), x(t - 6), x(t - 12)$  and  $(t - 18)$ . 1000 sample points are used in our study. The first 500 data pairs of the series are used as train-



**Table 2**  
Comparison (RMSE and GIN) between EGP & parameter optimization algorithms for Mackey–Glass problem (NFEs = 6,000,000).

| Hybrid algorithm     | RMSE training | RMSE testing | GIN     |
|----------------------|---------------|--------------|---------|
| FBBFNT_EGP&PSO       | 5.3000e-03    | 5.4000e-03   | 123,336 |
| FBBFNT_EGP&OPSO      | 5.5985e-04    | 5.8488e-04   | 63,608  |
| FBBFNT_EGP&ABC       | 8.2018e-04    | 8.8512e-04   | 70,911  |
| FBBFNT_EGP&ABC-OPSOP | 1.3529e-10    | 1.3534e-10   | 40,140  |

**Table 3**  
Comparison (NFEs and GIN) between EGP & parameter optimization algorithms for Mackey–Glass problem (RMSE of training = 1.00e-06, maximum GIN = 40,000).

| Hybrid algorithm     | NFEs      | GIN    | RMSE testing |
|----------------------|-----------|--------|--------------|
| FBBFNT_EGP&PSO       | 1,962,284 | 40,245 | 5.2000e-03   |
| FBBFNT_EGP&OPSO      | 3,954,397 | 40,001 | 2.9000e-03   |
| FBBFNT_EGP&ABC       | 4,101,431 | 40,785 | 3.6381e-03   |
| FBBFNT_EGP&ABC-OPSOP | 621,556   | 6231   | 6.8205e-06   |

ing data, while the remaining 500 are used to validate the model identified.

As we have seen in previous section that evolving FBBFNT depends essentially on two issues which are the FBBFNT structure's creation and optimization; and FBBFNT parameter's adjustment. For the structure optimization we apply the EGP and for the FBBFNT parameter's optimization, we essentially apply, in this study, three evolutionary computation algorithms: PSO [29], OPSO [42], ABC and the proposed ABC-OPSOP. We thus study the response of each algorithm for evolving FBBFNT.

In this case, we fix firstly the number of Function Evaluations (NFEs) to 6,000,000 and then we extract the RMSE values and the Global Iteration Number (GIN) for all parameter optimization algorithms.

Based on the initial parameter optimization setting values (Table 1), Table 2 makes a comparison between the different parameter optimization algorithms combined with the EGP (for the structure optimization).

We can observe from Table 2, that the combination between EGP and ABC-OPSOP for evolving FBBFNT performs better than the other combinations after 6,000,000 NFEs and provides good results by minimizing the prediction error.

Moreover, in order to make the comparison fair enough, we fix the RMSE of training equal to 1.00e-06. We then evaluate the number of Function Evaluations (NFEs) required by each of the FBBFNT parameter's competitor algorithms to reach the prescribed given error.

Table 3 shows, for all algorithms, the NFEs necessary to find the optimum solution previously fixed within a given tolerance or training RMSE. From simulations and results, it is remarkable that FBBFNT\_EGP&ABC-OPSOP algorithm reduces generally the NFEs to reach a given training RMSE value as compared with the other algorithms. We note, moreover, that FBBFNT\_EGP&PSO,

**Table 4**  
Comparison between the FBBFNT model and other systems for the Mackey–Glass time series.

| Method               | Solution quality      |                      | Convergence speed (GIN) | Complexity                      |                                       |
|----------------------|-----------------------|----------------------|-------------------------|---------------------------------|---------------------------------------|
|                      | Training error (RMSE) | Testing error (RMSE) |                         | Number of function hidden nodes | Number of function evaluations (NFEs) |
| PSO-BBFN [23]        | –                     | 2.7000e-02           | 20,000                  | 6 (1 hidden layer)              | –                                     |
| HMDDE-BBFNN [25]     | 9.4000e-03            | 1.7000e-02           | 10,000                  | 4 (1 hidden layer)              | –                                     |
| GA-BBFNN [24]        | –                     | 1.3000e-02           | –                       | 9 (1 hidden layer)              | –                                     |
| Classical RBF [38]   | 9.6000e-03            | 1.1400e-02           | –                       | 23 (1 hidden layer)             | –                                     |
| CPSO [39]            | 1.9900e-02            | 3.2200e-02           | –                       | 4 (1 hidden layer)              | –                                     |
| HCMSPSO [40]         | 9.5000e-03            | 2.0800e-02           | –                       | 4 (1 hidden layer)              | 15,000,000                            |
| FNT [26]             | 6.9000e-03            | 7.1000e-03           | –                       | 4 (2 hidden layers)             | –                                     |
| LNF [52]             | 7.0000e-04            | 7.9000e-03           | –                       | 6 (1 hidden layer)              | –                                     |
| FBBFNT_EGP&ABC-OPSOP | 9.9620e-07            | 2.0189e-06           | 7421                    | 2 (2 hidden layers)             | 865,199                               |

**Table 5**  
Comparison (RMSE and GIN) between EGP & parameter optimization algorithms for Jenkins–Box problem (NFEs = 6,000,000).

| Hybrid algorithm     | RMSE training | RMSE testing | GIN    |
|----------------------|---------------|--------------|--------|
| FBBFNT_EGP&PSO       | 0.01735       | 0.01814      | 42,163 |
| FBBFNT_EGP&OPSO      | 0.01245       | 0.01216      | 33,840 |
| FBBFNT_EGP&ABC       | 0.01471       | 0.01669      | 39,012 |
| FBBFNT_EGP&ABC-OPSOP | 0.01102       | 0.01202      | 20,840 |

**Table 6**  
Comparison (NFEs and GIN) between EGP & parameter optimization algorithms for Jenkins–Box problem (RMSE of training = 0.0125, maximum GIN = 40,000).

| Hybrid algorithm     | NFEs      | GIN    | RMSE testing |
|----------------------|-----------|--------|--------------|
| FBBFNT_EGP&PSO       | 2,101,145 | 42,754 | 0.02191      |
| FBBFNT_EGP&OPSO      | 1,381,078 | 16,200 | 0.01216      |
| FBBFNT_EGP&ABC       | 1,901,310 | 28,001 | 0.01962      |
| FBBFNT_EGP&ABC-OPSOP | 1,117,608 | 8754   | 0.01116      |

FBBFNT\_EGP&OPSO and FBBFNT\_EGP&ABC cannot even reach the error fixed in advance with the allowed number of iterations (GIN = 40,000).

To fully evaluate the performance of our new technique and the other models of the literature, a list of experimental tests were performed to predict Mackey–Glass time series.

The used Beta operator sets to create an optimal FBBFNT model is  $NS = F \cup T = \{\beta_2, \beta_3, /_3\} \cup \{x_1, x_2, x_3, x_4\}$ , where  $x_i$  ( $i = 1, 2, 3, 4$ ) denotes  $x(t)$ ,  $x(t - 6)$ ,  $x(t - 12)$  and  $x(t - 18)$ , respectively. After 20 global generations ( $G = 20$ ), 865,199 global number of function evaluations ( $NFEs = 865,199$ ), an optimal FBBFNT model was obtained with RMSE 9.9620e-07. The RMSE value for validation data set is 2.0189e-06. The evolved FBBFNT, and the actual time-series data and the output of FBBFNT model for training and testing samples are shown in Fig. 7.

In addition, the memory footprint of the source code is of the order of 10,720 Kb, knowing that the memory footprint of the input data is 40 Kb. For the execution time is of the order of 2924 s.

The proposed system is essentially compared with Hierarchical multi-dimensional differential evolution for the design of Beta basis function neural network (HMDDE-BBFNN) [25], the FNT model with Gaussian function as flexible neuron operator [26], the local least-squares support vector machines-based neuro-fuzzy model (LNF) [52] and also with other systems.

The HMDDE-BBFNN approach adopts for parameters: 50 for the population size, 10,000 for a total number of iterations, and 4 for the number of the hidden nodes. Moreover, the parameter settings of the FNT system [26] are 30 to the population size, 135 as generation number, and 4 as hidden function unit number (with two hidden layers). For LNF network, the authors use 6 neurons to generate their model.

A comparison result of different methods for forecasting Mackey–Glass data is shown in Table 4.

**Table 7**  
Comparison of training and testing errors of Box and Jenkins.

| Inputs           | Training error (RMSE) |                       |                  |                       | Testing error (RMSE) |                       |                  |                       |
|------------------|-----------------------|-----------------------|------------------|-----------------------|----------------------|-----------------------|------------------|-----------------------|
|                  | HMDDE [25]            | FBBFNT_EGP &OPSO [42] | FBBFNT_EGP & ABC | FBBFNT_EGP &ABC-OPSOP | HMDDE [25]           | FBBFNT_EGP &OPSO [42] | FBBFNT_EGP & ABC | FBBFNT_EGP &ABC-OPSOP |
| $y(t-1), u(t-3)$ | 0.1328                | 0.0034                | 0.0299           | <b>0.0023</b>         | 0.2276               | 0.0114                | 0.0269           | <b>0.0103</b>         |
| $y(t-3), u(t-4)$ | 0.0210                | 0.0060                | 0.0288           | <b>0.0040</b>         | 0.4224               | 0.0209                | 0.0267           | <b>0.0116</b>         |
| $y(t-2), u(t-4)$ | 0.1365                | 0.0057                | 0.0240           | <b>0.0045</b>         | 0.3200               | 0.0165                | 0.0223           | <b>0.0105</b>         |
| $y(t-1), u(t-2)$ | 0.1735                | 0.0057                | 0.0307           | <b>0.0047</b>         | 0.2334               | 0.0114                | 0.0295           | <b>0.0098</b>         |
| $y(t-1), u(t-4)$ | 0.2411                | 0.0047                | 0.0258           | <b>0.0040</b>         | 0.3745               | 0.0116                | 0.0245           | <b>0.0109</b>         |
| $y(t-4), u(t-4)$ | 0.1594                | 0.0071                | 0.0263           | <b>0.0066</b>         | 0.4549               | 0.0237                | 0.0268           | <b>0.0178</b>         |
| $y(t-2), u(t-3)$ | 0.1702                | 0.0066                | 0.0251           | <b>0.0056</b>         | 0.2700               | 0.0185                | 0.0271           | <b>0.0163</b>         |
| $y(t-1), u(t-1)$ | 0.1598                | 0.0124                | 0.0343           | <b>0.0095</b>         | 0.2577               | 0.0121                | 0.0427           | <b>0.0120</b>         |
| $y(t-4), u(t-3)$ | 0.1921                | 0.0142                | 0.0284           | <b>0.0102</b>         | 0.6148               | 0.0289                | 0.0334           | <b>0.0210</b>         |
| $y(t-1), u(t-6)$ | 0.6619                | 0.0124                | 0.0315           | <b>0.0113</b>         | 0.6638               | 0.0121                | 0.0248           | <b>0.0094</b>         |
| $y(t-3), u(t-3)$ | 0.1600                | 0.0106                | 0.0269           | <b>0.0097</b>         | 0.2521               | 0.0250                | 0.0321           | <b>0.0198</b>         |
| $y(t-2), u(t-2)$ | 0.2773                | 0.0157                | 0.0340           | <b>0.0118</b>         | 0.1615               | 0.0256                | 0.0324           | <b>0.0206</b>         |
| $y(t-1), u(t-5)$ | 0.3333                | 0.0124                | 0.0288           | <b>0.0091</b>         | 0.5595               | 0.0122                | 0.0259           | <b>0.0118</b>         |
| $y(t-4), u(t-5)$ | 0.0178                | 0.0116                | 0.0293           | <b>0.0090</b>         | 0.0203               | 0.0108                | 0.0256           | <b>0.0097</b>         |
| $y(t-2), u(t-1)$ | 0.1960                | 0.0236                | 0.0365           | <b>0.0183</b>         | 0.2759               | 0.0226                | 0.0377           | <b>0.0187</b>         |
| $y(t-2), u(t-5)$ | 0.2165                | 0.0236                | 0.0286           | <b>0.0117</b>         | 0.4021               | 0.0223                | 0.0226           | <b>0.0199</b>         |
| $y(t-3), u(t-5)$ | 0.1346                | 0.0137                | 0.0288           | <b>0.0118</b>         | 0.2307               | 0.0180                | 0.0256           | <b>0.0112</b>         |
| $y(t-3), u(t-2)$ | 0.2128                | 0.0326                | 0.0325           | <b>0.0177</b>         | 0.2760               | 0.0307                | 0.0350           | <b>0.0281</b>         |
| $y(t-4), u(t-6)$ | 0.1379                | 0.0209                | 0.0305           | <b>0.0169</b>         | 0.2635               | 0.0249                | 0.0261           | <b>0.0207</b>         |
| $y(t-2), u(t-6)$ | 0.2128                | 0.0236                | 0.0328           | <b>0.0209</b>         | 0.5590               | 0.0225                | 0.0323           | <b>0.0206</b>         |
| $y(t-4), u(t-2)$ | 0.2152                | 0.0229                | 0.0341           | <b>0.0165</b>         | 0.2737               | 0.0335                | 0.0383           | <b>0.0284</b>         |
| $y(t-3), u(t-6)$ | 0.2128                | 0.0330                | 0.0367           | <b>0.0208</b>         | 0.4027               | 0.0301                | 0.0256           | <b>0.0255</b>         |
| $y(t-3), u(t-1)$ | 0.2135                | 0.0326                | 0.0386           | <b>0.0233</b>         | 0.2803               | 0.0307                | 0.0378           | <b>0.0268</b>         |
| $y(t-4), u(t-1)$ | 0.2695                | 0.0398                | 0.0430           | <b>0.0332</b>         | 0.2695               | 0.0363                | 0.0409           | <b>0.0324</b>         |

**Table 8**  
Comparison (RMSE and GIN) between EGP & parameter optimization algorithms for sunspot number problem (NFEs = 600,000).

| Hybrid algorithm     | RMSE training | RMSE testing 1 | RMSE testing 2 | GIN    |
|----------------------|---------------|----------------|----------------|--------|
| FBBFNT_EGP&PSO       | 1.0138e-07    | 2.018e-07      | 2.113e-07      | 10,163 |
| FBBFNT_EGP&OPSO      | 1.2021e-09    | 2.077e-09      | 3.102e-09      | 15,840 |
| FBBFNT_EGP&ABC       | 1.0010e-08    | 1.067e-08      | 1.099e-08      | 18,481 |
| FBBFNT_EGP&ABC-OPSOP | 1.1981e-11    | 2.018e-11      | 2.113e-11      | 9432   |

**Table 9**  
Comparison (NFEs and GIN) between EGP & parameter optimization algorithms for sunspot number problem (RMSE of training = 1.00e-07, maximum GIN = 40,000).

| Hybrid algorithm     | NFEs    | GIN    | RMSE testing 1 | RMSE testing 2 |
|----------------------|---------|--------|----------------|----------------|
| FBBFNT_EGP&PSO       | 617,108 | 18,918 | 1.6078e-07     | 1.3128e-07     |
| FBBFNT_EGP&OPSO      | 784,763 | 17,281 | 7.3589e-08     | 6.0259e-08     |
| FBBFNT_EGP&ABC       | 801,904 | 18,002 | 1.2157e-07     | 1.5526e-03     |
| FBBFNT_EGP&ABC-OPSOP | 271,521 | 4502   | 3.0733e-08     | 2.9634e-08     |

It is clear from the results of our model and the errors provided by the other methods (Table 4), that our system gives the best prediction accuracy for the Mackey–Glass time-series. Our method can also minimize the number of hidden function units to two neurons in two hidden layers for EGP algorithm.

This result justifies more that we have seen in Section 3.2.4. In fact, the hybridization between OPSO and ABC improves the behavior of both algorithms.

For better explain these results theoretically, we are based, in the choice of metaheuristic algorithms for FBBFNT parameter optimization, on two essential phenomena: exploration and exploitation [55,56]. The exploration means the search for new solutions in the search space and the exploitation means the use and the improvement of existing solutions.

Indeed, PSO is known in the literature as a strong algorithm in the exploitation seen that it is based on a guided solution through the local and global optima. But it would be sometimes weak in exploration in the search space by getting stuck in a local optimum. For OPSO, it improves slightly the exploration compared to PSO but it can also get stuck in a local optimum because it doesn't use new solutions of the search space.

On the other hand, ABC rather promotes exploration as exploitation compared to other algorithms in the literature by seeking new solutions of the search space after a maximum number of trials.

For these reasons we hybridized both algorithms into a new algorithm (ABC-OPSOP) to benefit from their advantages. This strategy is also justified by the experimental results found in this section and the following sections.

#### 4.2. Example 2: Box and Jenkins' gas furnace problem

The gas furnace data of Box and Jenkins [41] was saved from a combustion process of a methane–air mixture. It is frequently used as a benchmark example for testing prediction algorithms. The data set forms of 296 pairs of input–output measurements. The input  $u(t)$  is the gas flow into the furnace and the output  $y(t)$  is the CO<sub>2</sub> concentration in outlet gas.

The inputs for constructing FBBFNT model are  $y(t-1)$ ,  $u(t-4)$ , and the output is  $y(t)$ . In this work, 200 data samples are used for training and the remaining data samples are used for testing the performance of the evolved model.

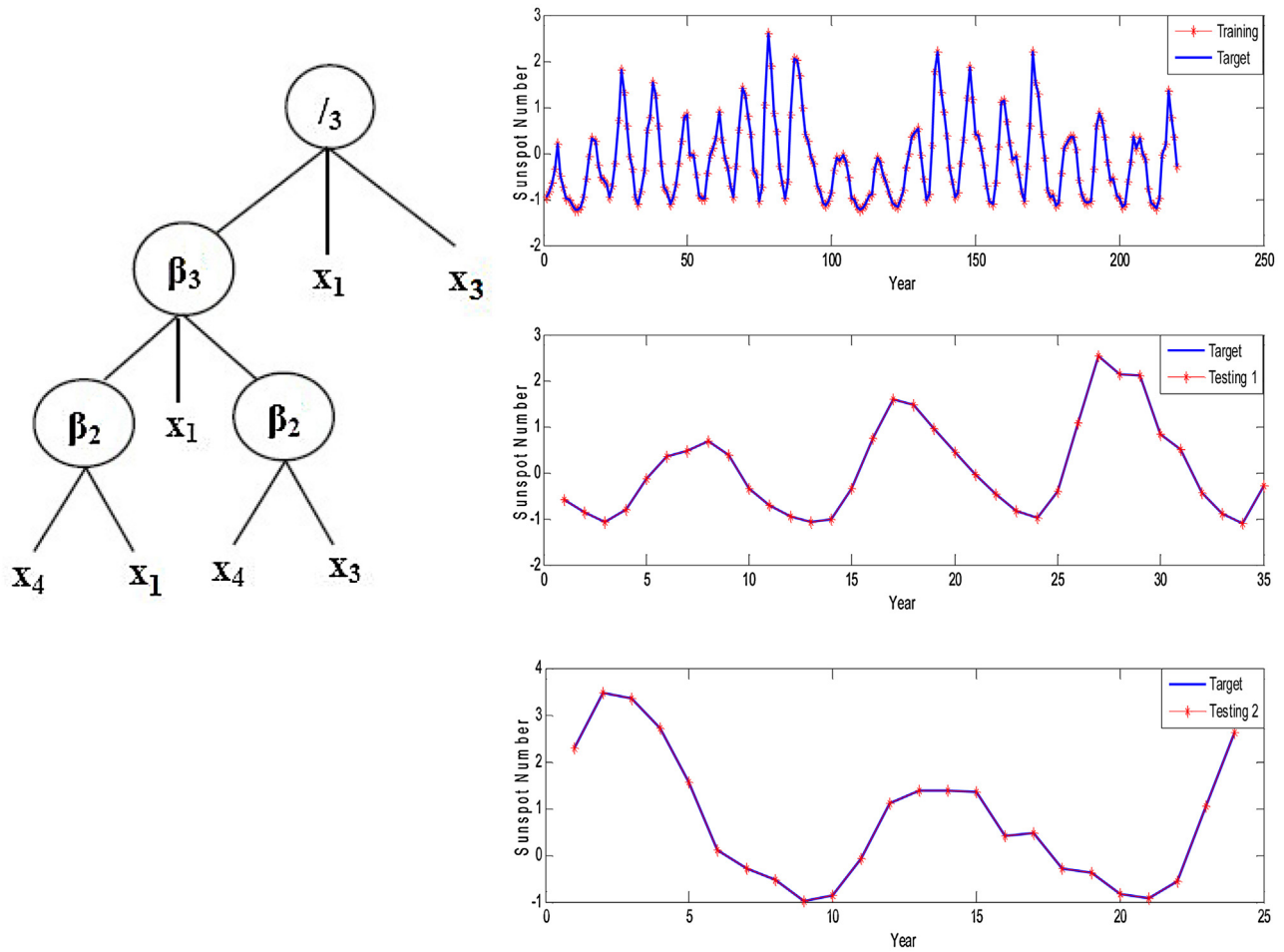


Fig. 9. The evolved FBBFNT (left), and the actual time series data and the output of the FBBFNT model for training, test 1 and test 2 to predict the sunspot number time-series.

Table 10 Comparison between the FBBFNT model and other systems for the sunspot number time-series.

| Method               | Solution quality |                |                | GIN  | Complexity                      |         |
|----------------------|------------------|----------------|----------------|------|---------------------------------|---------|
|                      | RMSE training    | RMSE testing 1 | RMSE testing 2 |      | Number of function hidden nodes | NFEs    |
| Recurrent net [44]   | 2.6790e-01       | 3.1510e-01     | 9.0050e-01     | –    | 2 (1 hidden layer)              | –       |
| RFNN [45]            | –                | 2.7490e-01     | 6.2490e-01     | –    | 7 (1 hidden layer)              | –       |
| FENN [47]            | –                | –              | 5.7850e-01     | –    | –                               | –       |
| FWNN-S [48]          | 2.5270e-01       | 3.3410e-01     | 5.2990e-01     | 200  | 16 rules                        | –       |
| FWNN-R [48]          | 2.3830e-01       | 3.3500e-01     | 6.8850e-01     | 200  | 16 rules                        | –       |
| FWNN-M [48]          | 2.4300e-01       | 3.1520e-01     | 6.0800e-01     | 200  | 16 rules                        | –       |
| ABC_BBFNN [46]       | 1.2000e-03       | 1.8000e-03     | 4.4000e-03     | –    | –                               | –       |
| LNF[52]              | 1.8880e-03       | 2.5370e-03     | 3.8080e-03     | –    | 7 (1 hidden layer)              | –       |
| FBBFNT_EGP&ABC-OPSOP | 3.1035e-08       | 7.2848e-07     | 8.0029e-07     | 3821 | 3 (2 hidden layers)             | 631,075 |

Table 11 Comparison (RMSE and GIN) between EGP & parameter optimization algorithms for Lorenz problem (NFEs = 200,000).

| Hybrid algorithm     | RMSE training | RMSE testing | GIN  |
|----------------------|---------------|--------------|------|
| FBBFNT_EGP&PSO       | 3.3552e-03    | 8.3352e-03   | 3386 |
| FBBFNT_EGP&OPSO      | 6.0945e-04    | 8.6922e-04   | 1798 |
| FBBFNT_EGP&ABC       | 3.7193e-04    | 5.8124e-04   | 2931 |
| FBBFNT_EGP&ABC-OPSOP | 4.6529e-09    | 1.3534e-08   | 885  |

Table 12 Comparison (NFEs and GIN) between EGP & parameter optimization algorithms for Lorenz problem (RMSE of training = 1.00e-09, maximum GIN = 40,000).

| Hybrid algorithm     | NFEs    | GIN    | RMSE testing |
|----------------------|---------|--------|--------------|
| FBBFNT_EGP&PSO       | 607,883 | 12,734 | 8.3152e-09   |
| FBBFNT_EGP&OPSO      | 642,542 | 8772   | 5.5060e-09   |
| FBBFNT_EGP&ABC       | 700,092 | 10,041 | 7.9681e-09   |
| FBBFNT_EGP&ABC-OPSOP | 266,106 | 4477   | 5.0145e-10   |

In the first case, we fix the number of Function Evaluations (NFEs) to 6,000,000 and we extract the RMSE values and the Global Iteration Number (GIN) for all parameter optimization algorithms. So Table 5 makes a comparison between the different parameter

optimization algorithms combined with EGP (for the architecture optimization).

We can notice, from Table 5, that there is not a significant difference between the different algorithm responses. However, the combination between EGP and ABC-OPSOP for evolving FBBFNT rel-

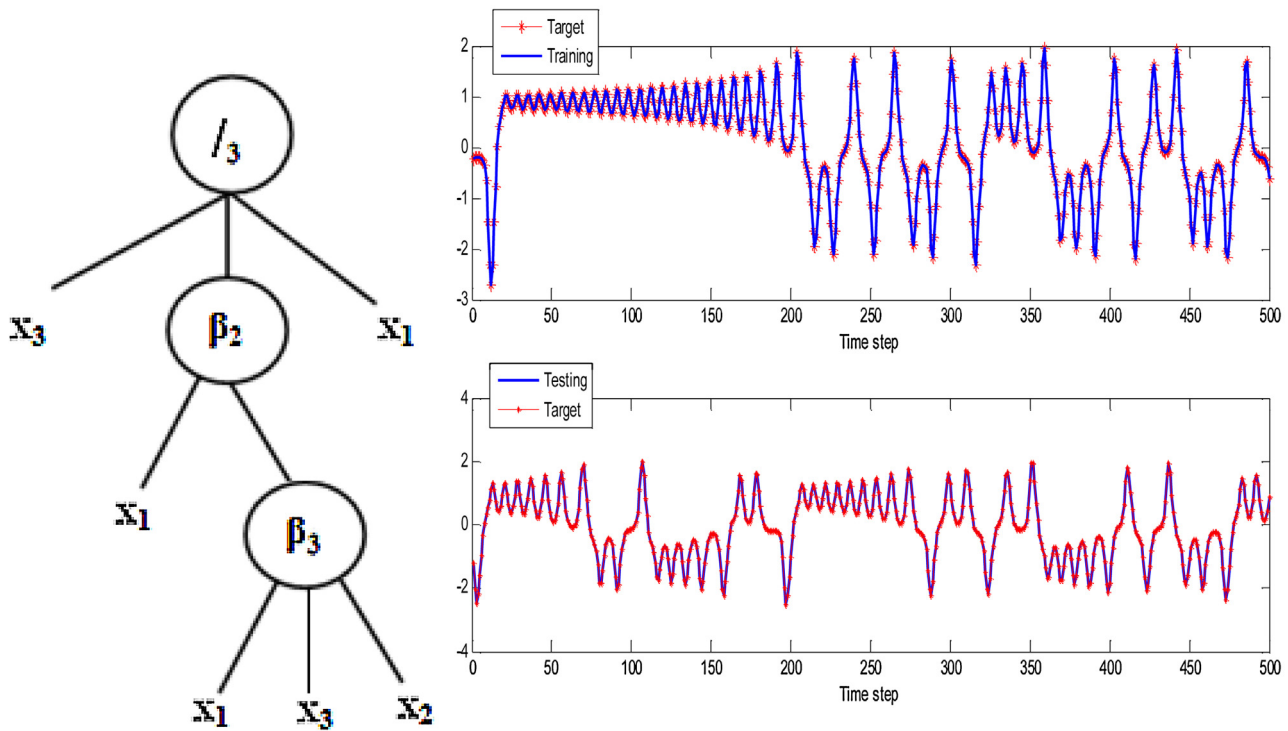


Fig. 10. The evolved FBBFNT, and the actual time series data and the output of the FBBFNT model for training and test examples to predict the Lorenz chaotic time-series.

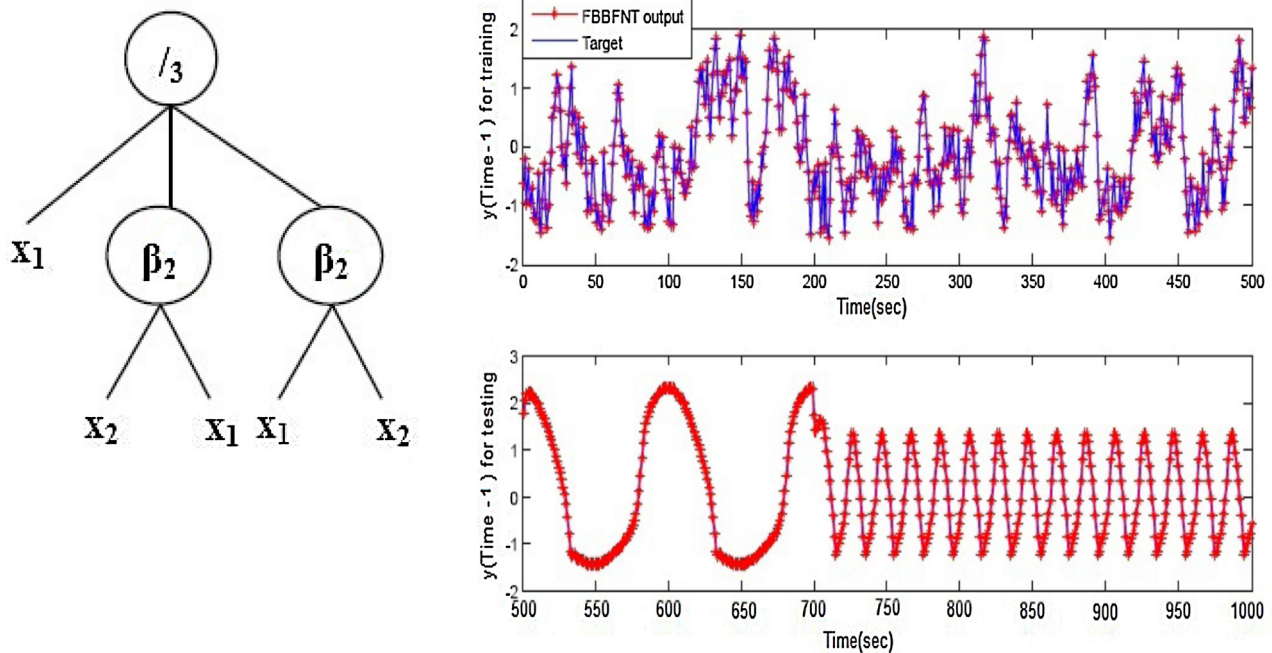


Fig. 11. The evolved FBBFNT, and the actual time series data and the output of the FBBFNT model for training and test examples to identify the nonlinear plant identification.

actively performs better than the other combinations after 6,000,000 NFEs, in terms of accuracy and also in term of global iteration number.

In the second case, we fix the RMSE of training equal to 0.0125. We then extract the number of Function Evaluations NFEs required by each of the FBBFNT parameter's competitor algorithms to reach the prescribed given error.

Table 6 shows, for all algorithms, the NFEs necessary to find the optimum solution previously fixed within a given tolerance or

RMSE training. From simulations and results, it is remarkable that FBBFNT\_EGP&ABC-OPSOP algorithm reduces significantly the NFEs to reach a given training RMSE value as compared with the other algorithms. We note, moreover, that FBBFNT\_EGP&PSO cannot even reach the error fixed in advance at the allowed number of iterations (GIN = 40,000).

To observe the behavior of FBBFNT\_EGP&ABC-OPSOP, the used instruction set for creating a FBBFNT model is  $S = F \cup T = \{\beta_2, \beta_3, \beta_4, /_4\} \cup \{x_1, x_2\}$ , where  $x_i$  ( $i=1, 2$ ) denotes

**Table 13**  
Comparison between the FBBFNT model and other systems for the Lorenz time-series.

| Method               | Solution quality      |                      | GIN  | Complexity                      |         |
|----------------------|-----------------------|----------------------|------|---------------------------------|---------|
|                      | Training error (RMSE) | Testing error (RMSE) |      | Number of function hidden nodes | NFEs    |
| UD-SVM [50]          | –                     | 2.9000e-01           | –    | –                               | –       |
| MLP-EKF [51]         | 1.4500e-02            | 4.0800e-02           | 1000 | 37 (3 hidden layers)            | –       |
| MLP-BLM [51]         | 1.7400e-02            | 3.1400e-02           | 1000 | 33 (3 hidden layers)            | –       |
| RNN-BPTT [51]        | 2.2700e-02            | 4.3600e-02           | 1000 | 19 (3 hidden layers)            | –       |
| RNN-RTRL [51]        | 2.2900e-02            | 4.2000e-02           | 1000 | 19 (3 hidden layers)            | –       |
| RNN-EKF [51]         | 1.8200e-02            | 3.5200e-02           | 1000 | 19 (3 hidden layers)            | –       |
| RBLM-RNN [51]        | 1.8200e-02            | 3.0400e-02           | 1000 | 16 (3 hidden layers)            | –       |
| LNF [52]             | 3.9000e-03            | 8.1000e-03           | –    | –                               | –       |
| FBBFNT_EGP&ABC-OPSOP | 7.4320e-08            | 1.0218e-07           | 3872 | 2 (2 hidden layers)             | 204,911 |

**Table 14**  
Comparison (RMSE and GIN) between EGP & parameter optimization algorithms for nonlinear plan identification (NFEs = 200,000).

| Hybrid algorithm     | RMSE training | RMSE testing | GIN    |
|----------------------|---------------|--------------|--------|
| FBBFNT_EGP&PSO       | 2.3452e-04    | 6.3362e-04   | 7386   |
| FBBFNT_EGP&OPSO      | 3.1645e-06    | 5.6822e-06   | 10,098 |
| FBBFNT_EGP&ABC       | 5.0319e-06    | 8.0077e-06   | 11,123 |
| FBBFNT_EGP&ABC-OPSOP | 4.7509e-08    | 6.0531e-08   | 9485   |

**Table 15**  
Comparison (NFEs and GIN) between EGP & parameter optimization algorithms for nonlinear plan identification (RMSE of training = 1.00e-10, maximum GIN = 40,000).

| Hybrid algorithm     | NFEs      | GIN    | RMSE testing |
|----------------------|-----------|--------|--------------|
| FBBFNT_EGP&PSO       | 6,073,307 | 40,230 | 4.3392e-09   |
| FBBFNT_EGP&OPSO      | 1,235,763 | 14,662 | 3.9081e-10   |
| FBBFNT_EGP&ABC       | 4,651,800 | 17,002 | 5.8644e-10   |
| FBBFNT_EGP&ABC-OPSOP | 732,933   | 31,752 | 1.4082e-10   |

$y(t - 1)$ ,  $u(t - 4)$ , respectively. After 12 generations of the learning algorithm ( $G = 12$ ), 450,978 global number of function evaluations (NFEs = 450,978), the optimal Beta basis function neural tree model was obtained with the RMSE 0.004096. The RMSE value for validation data set is 0.010918. The evolved FBBFNT, and the actual time-series data and the output of FBBFNT model for training and testing samples are shown in Fig. 8. In addition, the memory footprint of the source code is of the order of 2508 Kb, knowing that the memory footprint of the input data is 7.008 Kb. The execution time is of the order of 1021 s.

Furthermore, we have taken two inputs the first one is from furnace output and other is from furnace input. Therefore we have construct 24 models of different input–output and the training and testing performances of these models are given in Table 7. From the simulation results of Table 7, it can be seen that the proposed FBBFNT\_EGP&ABC-OPSOP system works well for generating prediction models of Box and Jenkins’ gas furnace problem.

4.3. Example 3: prediction of sunspot number time series

This example presents the series of the sunspot annual average numbers which show the yearly average relative number of sunspot observed [43]. The sunspot number time series is considered as a real-world highly-complex and non-stationary time series. It is

**Table 16**  
Comparison between the FBBFNT model and other systems for the nonlinear plant identification.

| Method               | Solution quality      |                      | GIN   | Complexity                      |         |
|----------------------|-----------------------|----------------------|-------|---------------------------------|---------|
|                      | Training error (RMSE) | Testing error (RMSE) |       | Number of function hidden nodes | NFEs    |
| ODE [61]             | 1.9000e-02            | 1.1370e-01           | 1000  | 11 (1 hidden layer)             | –       |
| HMDDE [25]           | 1.9000e-02            | 1.1000e-01           | 10000 | 3 (1 hidden layer)              | –       |
| FBBFNT_EGP&ABC-OPSOP | 1.8961e-10            | 2.1203e-10           | 15067 | 2 (1 hidden layer)              | 270,311 |

recorded for the years 1700–1979. The dataset is available at the National Geophysical Data Center website (<http://www.ngdc.noaa.gov/stp/solar/ssndata.html>).

To make our comparisons meaningful with related works [43–45], the dataset is divided into three parts. The data points between 1700 and 1920 are used for training FBBFNT model. For the test, two sets are used; the first one is from 1921 to 1955 and the second is from 1956 to 1979. The  $y(t - 4)$ ,  $y(t - 3)$ ,  $y(t - 2)$  and  $y(t - 1)$  are used as inputs to the FBBFNT model in order to predict the output  $y(t)$ .

For FBBFNT’s structure optimization, we apply the Extended Genetic Programming and for the parameter’s adjustment we test PSO, OPPO, and ABC-OPSOP algorithms.

Firstly, we fix the number of Function Evaluations (NFEs) to 600,000 and we illustrate the RMSE values and the Global Iteration Number (GIN) for all parameter optimization algorithms.

As presented in Table 8, a comparison is performed over a diverse collection of the different combinations between parameter optimization algorithms and EGP.

From Table 8, it is clear that the combination between EGP and ABC-OPSOP for evolving FBBFNT is having less training and testing errors in comparison to the other combinations after 600,000 NFEs.

In order to compare the convergence speed and the evolving FBBFNT complexity of the different algorithms, several numbers of GIN and NFEs were noted. In Table 9, we fixed the error value equal to 1:00e-07, then we evaluate both GIN and NFEs. Table 9 makes a comparison between the different parameter optimization algorithms.

It is clear from Table 9 that ABC-OPSOP algorithm reduces generally the NFEs to reach a given training RMSE value as compared with the other algorithms.

After that, the accuracy and the efficiency of the proposed methodology were demonstrated through several comparisons with other available learning approaches developed in recent years.

The used Beta operator sets to create an optimal FBBFNT\_EGP&ABC-OPSOP model is  $S = F \cup T = \{\beta_2, \beta_3, /_3\} \cup \{x_1, x_2, x_3, x_4\}$ , where  $x_i$  ( $i = 1, 2, 3, 4$ ) denotes  $y(t - 4)$ ,  $y(t - 3)$ ,  $y(t - 2)$  and  $y(t - 1)$ , respectively.

After 22 generations of the evolution ( $G = 22$ ), 631,075 global number of function evaluations (NFEs = 631,075), an optimal FBBFNT model was obtained with RMSE 3.1035e-08. The RMSE value for the first data set validation is 7.2848e-07 and for the sec-

ond data set validation is  $8.0029e-07$ . The evolved FBBFNT, and the actual time-series data and the output of FBBFNT model for training and the two test cases are shown in Fig. 9.

Table 10 illustrates the comparison of the proposed algorithm with other models according to the training and testing errors. As evident from Table 10, FBBFNT\_EGP&ABC-OPSOP shows again the efficiencies considering the compromise between solution quality and complexity for the sunspot number time series.

#### 4.4. Example 4: Lorenz chaotic time series prediction

The Lorenz system [49] is a model of fluid motion between a hot surface and a cool surface. It is described by the following nonlinear ordinary differential equations:

$$\begin{cases} \dot{x} &= \sigma(y - x) \\ \dot{y} &= -y - xz + rx \\ \dot{z} &= xy - bz \end{cases} \quad (17)$$

In this example, the  $x$ -component in the Lorenz equations is used as the time series. The data that describe the Lorenz attractor, were generated by solving the system of differential equations, with  $\sigma=10$ ,  $r=28$  and  $b=8/3$ . The data were used as inputs to the neural networks. The prediction is based on four past values ( $x(t-4)$ ,  $x(t-3)$ ,  $x(t-2)$ ,  $x(t-1)$ ) and thus the output pattern is  $x(t) = f(x(t-4), x(t-3), x(t-2), x(t-1))$ .

To study the Lorenz chaotic problem, we choose EGP for the FBBFNT's structure optimization and we vary the algorithms of FBBFNT's parameter optimization.

In the first case, we fix the number of Function Evaluations (NFEs) to 200,000 and we extract the RMSE values and the Global Iteration Number (GIN) for all parameter optimization algorithms.

Based on the initial parameter optimization setting values (Table 1), Table 11 makes a comparison between the different proposed parameter optimization algorithms combined with EGP.

We can observe from Table 11, that the combination between EGP and ABC-OPSOP for evolving FBBFNT performs better than the other combinations after 200,000 NFEs and provides good results by generating high accuracy system.

In the second case, we fix the RMSE of training to  $1.00e-09$ . We then extract the number of Function Evaluations NFEs required by each of the FBBFNT parameter's competitor algorithms to reach the prescribed given error. Table 12 illustrates a comparison between the different parameter optimization algorithms combined with EGP.

It is remarkable from Table 12 that FBBFNT\_EGP&ABC-OPSOP algorithm reduces generally the NFEs to reach a given training RMSE value as compared with the other algorithms.

To fully evaluate the performance of our new technique and the other models of the literature, a list of experimental tests were performed for predicting Lorenz time series.

The used Beta operator sets to create an optimal FBBFNT model is  $S = F \cup T = \{\beta_2, \beta_3, /_3\} \cup \{x_1, x_2, x_3, x_4\}$ , where  $x_i$  ( $i=1, 2, 3, 4$ ) denotes  $x(t-4)$ ,  $x(t-3)$ ,  $x(t-2)$  and  $x(t-1)$ , respectively.

After 19 generations of the evolution ( $G=19$ ), 204,911 global number of function evaluations (NFEs=204,911), an optimal FBBFNT model was obtained with RMSE  $7.4320e-08$ . The RMSE value for validation data set is  $1.0218e-07$ . In addition, the memory footprint of the source code is of the order of 2480 Kb, knowing that the memory footprint of the input data is 40 Kb. The execution time is of the order of 979 s. The evolved FBBFNT, and the actual time-series data and the output of FBBFNT model for training and testing examples are shown in Fig. 10.

Table 13 illustrates the comparison of the proposed algorithm with other models according to the training and testing errors. This

table shows the performance of our approach for Lorenz chaotic time series comparing with the other model.

#### 4.5. Example 5: nonlinear plant identification

In this example, the nonlinear system [41] to be identified is expressed by:

$$y_p(t+1) = \frac{y_p(t)[y_p(t-1)+2][y_p(t)+2.5]}{8.5+[y_p(t)]^2+[y_p(t-1)]^2} + u(t) \quad (18)$$

where  $y_p(t)$  is the output of the system at the  $t$ th time step and  $u(t)$  is the plant input which is uniformly bounded in the region  $[-2,2]$ . The adopted identification model is as the following form:

$$y_{pi}(t+1) = f(y_p(t), y_p(t-1)) + u(t) \quad (19)$$

where  $f(y_p(t), y_p(t-1))$  is the nonlinear function of  $y_p(t)$  and  $y_p(t-1)$  that will be input of our model and related works; and  $y_{pi}(t+1)$  will be the output from the neural models.

The objective is to train the artificial neural networks used to identify the above system such that when an input  $u(t)$  is presented to the network and to the nonlinear identification system, the network outputs  $y_{pi}(t+1)$  and the actual system output  $y_p(t+1)$  should be as close as possible i.e., the identified system output should follow the actual system output.

In this example, 500 data samples are used for training and 500 data samples are used for testing the performance of the evolved model. After the training is over, the identifier's prediction ability has been tested for the input calculated as following:

$$u(t) = \begin{cases} 2 \cos\left(\frac{2\pi t}{100}\right) & \text{if } t \leq 200 \\ 1.2 \sin\left(\frac{2\pi t}{20}\right) & \text{if } 200 < t \leq 500 \end{cases} \quad (20)$$

In the first case, we fix the number of Function Evaluations (NFEs) to 200,000 and we extract the RMSE values and the Global Iteration Number (GIN) for all parameter optimization algorithms.

Table 14 illustrates a comparison between the different parameter optimization algorithms combined with EGP.

We can observe from Table 14, that the combination between EGP and ABC-OPSOP for evolving FBBFNT performs better than the other combinations after 200,000 NFEs and provides good results by minimizing the prediction error.

In order to compare the convergence speed and the evolving FBBFNT complexity of the different algorithms, several numbers of GIN and NFEs were noted. In Table 15, we fixed the error value equal to  $1.00e-10$ , and then we evaluate both GIN and NFEs. This table makes a comparison between the different adopted parameter optimization algorithms combined with EGP (for the structure optimization).

The FBBFNT\_EGP&ABC-OPSOP hybrid system gives relatively the best results for the number of function evaluations as well as for the testing error. This comparison is illustrated in Table 16.

To fully evaluate the performance of our new technique and the other models of the literature, a list of experimental tests were performed for identifying nonlinear plant identification.

From the previous cited combinations, we choose the FBBFNT\_EGP&ABC-OPSOP as an example to detail it. In fact, the used Beta basis function sets to create an optimal FBBFNT model with EGP&ABC-OPSOP system is  $NS = F \cup T = \{\beta_2, \beta_3, /_3\} \cup \{x_1, x_2\}$ , where  $x_i$  ( $i=1, 2$ ) denotes  $y_p(t)$  and  $y_p(t-1)$ , respectively.

After 10 global generations ( $G=10$ ), 270,311 global number of function evaluations, and 8009 global iterations of the hybrid learning algorithm, an optimal FBBFNT model was obtained with RMSE  $1.8961e-10$ . The RMSE value for identification data set is  $2.1203e-10$ .

In addition, the memory footprint of the source code is of the order of 97921 Kb, knowing that the memory footprint of the input data is 30Kb. The execution time is of the order of 3016s. The evolved FBBFNT, and the nonlinear plant identification and the output of FBBFNT model for training and testing examples are shown in Fig. 11.

According to the available measures of other methods, we can conclude from the comparison table, that the FBBFNT\_EGP&ABC-OPSOP model is relatively more efficient considering the compromise between solution quality and complexity, for the four tested time series as well as for the nonlinear plant identification.

## 5. Conclusion

In this paper, a hybrid learning algorithm is proposed to create and evolve a flexible beta basis function neural tree (FBBFNT) model for the prediction of benchmark problems. As evident from the experiments, the new learning algorithm can successfully optimize the structure and parameters of Beta basis function neural network simultaneously based on the tree representation. In fact, the FBBFNT structure is developed using Extended Genetic Programming (EGP) and the Beta parameters and connected weights are optimized by the Artificial Bee Colony algorithm modified by Opposite-based Particle Swarm Optimization positions (ABC-OPSOP).

The results show that the FBBFNT\_EGP&ABC-OPSOP model can effectively predict time-series problems (Mackey–Glass chaotic, the Jenkins–Box, sunspot number and Lorenz chaotic time series) and identify an example of nonlinear plant identification.

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